NAG C Library Function Document

nag_dsptrs (f07pec)

1 Purpose

nag_dsptrs (f07pec) solves a real symmetric indefinite system of linear equations with multiple right-hand sides, $AX = B$, where A has been factorized by nag_dsptrf (f07pdc), using packed storage.

2 Specification

```
void nag_dsptrs (Nag_OrderType order, Nag_UploType uplo, Integer n, Integer nrhs,
     const double ap[], const Integer ipiv[], double b[], Integer pdb,
     NagError *fail)
```

3 Description

To solve a real symmetric indefinite system of linear equations $AX = B$, this function must be preceded by a call to nag_dsptrf (f07pdc) which computes the Bunch–Kaufman factorization of A using packed storage.

If **uplo** = **Nag_Upper**, $A = PUDU^TP^T$, where P is a permutation matrix, U is an upper triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 blocks; the solution X is computed by solving $PUDY = B$ and then $U^TP^TX = Y$.

If **uplo** = **Nag_Lower**, $A = PLDL^TP^T$, where L is a lower triangular matrix; the solution X is computed by solving $PLDY = B$ and then $L^TP^TX = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates how A has been factorized as follows:

if **uplo** = **Nag_Upper**, $A = PUDU^TP^T$, where U is upper triangular;

if **uplo** = **Nag_Lower**, $A = PLDL^TP^T$, where L is lower triangular.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **nrhs** – Integer *Input*

On entry: r, the number of right-hand sides.

Constraint: **nrhs** ≥ 0.

5: **ap**[dim] – const double *Input*

Note: the dimension, dim, of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$.

On entry: details of the factorization of A stored in packed form, as returned by nag_dsptrf (f07pdc).

6: **ipiv**[dim] – const Integer *Input*

Note: the dimension, dim, of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

On entry: details of the interchanges and the block structure of D, as returned by nag_dsptrf (f07pdc).

7: **b**[dim] – double *Input/Output*

Note: the dimension, dim, of the array **b** must be at least $\max(1, \mathbf{pdb} \times \mathbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \mathbf{pdb} \times \mathbf{n})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (i, j)th element of the matrix B is stored in $\mathbf{b}[(j-1) \times \mathbf{pdb} + i - 1]$ and if **order** = **Nag_RowMajor**, the (i, j)th element of the matrix B is stored in $\mathbf{b}[(i-1) \times \mathbf{pdb} + j - 1]$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

8: **pdb** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = **Nag_ColMajor**, **pdb** $\geq \max(1, \mathbf{n})$;
if **order** = **Nag_RowMajor**, **pdb** $\geq \max(1, \mathbf{nrhs})$.

9: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

On entry, **nrhs** = ⟨*value*⟩.
Constraint: **nrhs** ≥ 0.

On entry, **pdb** = ⟨*value*⟩.
Constraint: **pdb** > 0.

NE_INT_2

On entry, **pdb** = ⟨*value*⟩, **n** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = ⟨*value*⟩, **nrhs** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x=b$, where

if **uplo** $=$ **Nag_Upper**, $|E| \leq c(n)\epsilon P|U|\,|D|\,|U^T|P^T$;

if **uplo** $=$ **Nag_Lower**, $|E| \leq c(n)\epsilon P|L|\,|D|\,|L^T|P^T$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \||A^{-1}|\,|A|\,|x|\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \||A^{-1}|\,|A|\|_\infty \leq \kappa_\infty(A)$. Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_dsprfs (f07phc), and an estimate for $\kappa_\infty(A)$ $(= \kappa_1(A))$ can be obtained by calling nag_dspcon (f07pgc).

8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This function may be followed by a call to nag_dsprfs (f07phc) to refine the solution and return an error estimate.

The complex analogues of this function are nag_zhptrs (f07psc) for Hermitian matrices and nag_zsptrs (f07qsc) for symmetric matrices.

9 Example

To solve the system of equations $AX=B$, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix} \quad and \quad B = \begin{pmatrix} -9.50 & 27.85 \\ -8.38 & 9.90 \\ -6.07 & 19.25 \\ -0.96 & 3.93 \end{pmatrix}.$$

Here A is symmetric indefinite, stored in packed form, and must first be factorized by nag_dsptrf (f07pdc).

9.1 Program Text

```
/* nag_dsptrs (f07pec) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */
```

```
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer  ap_len, i, j, n, nrhs, pdb;
  Integer  exit_status=0;
  NagError fail;
  Nag_UploType  uplo_enum;
  Nag_OrderType order;
  /* Arrays */
  Integer *ipiv=0;
  char    uplo[2];
  double  *ap=0, *b=0;

#ifdef NAG_COLUMN_MAJOR
#define A_UPPER(I,J) ap[J*(J-1)/2 + I - 1]
#define A_LOWER(I,J) ap[(2*n-J)*(J-1)/2 + I - 1]
#define B(I,J) b[(J-1)*pdb + I - 1]
  order = Nag_ColMajor;
#else
#define A_LOWER(I,J) ap[I*(I-1)/2 + J - 1]
#define A_UPPER(I,J) ap[(2*n-I)*(I-1)/2 + J - 1]
#define B(I,J) b[(I-1)*pdb + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f07pec Example Program Results\n\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%ld%*[^\n] ", &n, &nrhs);
  ap_len = n*(n+1)/2;
#ifdef NAG_COLUMN_MAJOR
  pdb = n;
#else
  pdb = nrhs;
#endif

  /* Allocate memory */
  if ( !(ap = NAG_ALLOC(ap_len, double)) ||
       !(ipiv = NAG_ALLOC(n, Integer)) ||
       !(b = NAG_ALLOC(n * nrhs, double)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A and B from data file */
  Vscanf(" ' %1s '%*[^\n] ", uplo);
  if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
  else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
  else
    {
      Vprintf("Unrecognised character for Nag_UploType type\n");
      exit_status = -1;
      goto END;
    }
  if (uplo_enum == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= n; ++j)
```

```
            Vscanf("%lf", &A_UPPER(i,j));
        }
      Vscanf("%*[^\n] ");
    }
  else
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = 1; j <= i; ++j)
            Vscanf("%lf", &A_LOWER(i,j));
        }
      Vscanf("%*[^\n] ");
    }
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= nrhs; ++j)
        Vscanf("%lf", &B(i,j));
    }
  Vscanf("%*[^\n] ");

  /* Factorize A */
  f07pdc(order, uplo_enum, n, ap, ipiv, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07pdc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Compute solution */
  f07pec(order, uplo_enum, n, nrhs, ap, ipiv, b, pdb, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07pec.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Print solution */
  x04cac(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs, b, pdb,
         "Solution(s)", 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from x04cac.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
 END:
  if (ap) NAG_FREE(ap);
  if (ipiv) NAG_FREE(ipiv);
  if (b) NAG_FREE(b);
  return exit_status;
}
```

9.2 Program Data

```
f07pec Example Program Data
  4  2                         :Values of N and NRHS
  'L'                          :Value of UPLO
  2.07
  3.87  -0.21
  4.20   1.87   1.15
 -1.15   0.63   2.06  -1.81   :End of matrix A
 -9.50  27.85
 -8.38   9.90
 -6.07  19.25
 -0.96   3.93                 :End of matrix B
```

9.3 Program Results

```
f07pec Example Program Results

 Solution(s)
            1           2
 1     -4.0000      1.0000
 2     -1.0000      4.0000
 3      2.0000      3.0000
 4      5.0000      2.0000
```